**Financial volatility modeling: The feedback asymmetric conditional autoregressive range model**

Dmitri Teller, Elina Bukhanov

August 16, 2019

**Contents**

[**1.** **Introduction** 2](#_Toc16965145)

[**1.1.** **Background** 2](#_Toc16965146)

[**1.2.** **The problem** 2](#_Toc16965147)

[**2.** **Model and assumptions** 2](#_Toc16965148)

[**2.1.** **FCARR Model assumptions** 2](#_Toc16965149)

[**2.2.** **The reason for FACARR model assumption** 2](#_Toc16965150)

[**2.3.** **ACARR model and its implied assumption** 3](#_Toc16965151)

[**2.4.** **Feedback ACARR model (FACARR)** 3](#_Toc16965152)

[**2.5.** **What if the FCARR assumptions are removed?** 4](#_Toc16965153)

[**2.6.** **What data is needed to be collected to test the FCARR model?** 4](#_Toc16965154)

[**3.** **Empirical studies** 4](#_Toc16965155)

[**3.1.** **Granger causality testing results** 4](#_Toc16965156)

[**3.2.** **In-sample and out-of-sample forecasts** 4](#_Toc16965157)

[**4.** **Conclusion** 5](#_Toc16965158)

[**4.1.** **Conclusions of the authors** 5](#_Toc16965159)

[**4.2.** **How according to the authors the model can be improved?** 5](#_Toc16965160)

1. **Introduction**
   1. **Background**

For a long time, a high – low price range has been used as an efficient volatility estimator in many works, but in 2013 Degiannakis and Livada revealed that the range-based volatility estimator is more accurate.

There were different models describing the price range dynamics. In the beginning all of those models were symmetric, the maximum price and the minimum price were processed in a symmetric way. For this case Chou proposed the conditional autoregressive range (CARR) model in 2005. It was found that it was a worthy candidate in volatility modeling in comparison with the well-known generalized autoregressive conditional heteroskedasticity (GARCH) model.

The asymmetric conditional autoregressive range (ACARR) model was proposed by Chou in 2006. This model gave an opportunity to treat the dynamics of the upward range and the downward range in a different, asymmetric way. The main assumption is that the upward range and the downward range move separately.

* 1. **The problem**

The next idea was that the assumption of the ACARR model is not right for a few reasons:

* Firstly, investors use all the available information and not only the history upward (downward) range information to forecast the future upward (downward) range.
* Secondly, there is a market overreaction that implies that upward (downward) range helps to predict the downward (upward) range. This idea was tested by the Granger causality test and it was found a significant cross-interdependence between the upward range and the downward range.

Thus, to improve the ACARR model was proposed the feedback asymmetric conditional autoregressive range (henceforth FACARR) model that explains this cross-interdependence.

1. **Model and assumptions**
   1. **FCARR Model assumptions**

Significant cross-interdependence between the upward range and the downward range.

* 1. **The reason for FACARR model assumption**

Improving the ACARR model, testing and disproof the main ACARR assumption that UPR/DWNR ranges are independent.

* 1. **ACARR model and its implied assumption**

The conditional mean of the upward (downward) range in the ACARR model can be submitted as follows:

= *u* or *d*,

= or ,

is the lag operator,

=

From equation 1 we can see that the conditional mean of the upward range (downward range) in the ACARR model is specified to be fully determined by its historical upward range (downward range) information. This assumption involves independence between the upward range and the downward range. Since this is the main assumption of the ACARR model, it should be treated carefully, because if it is not right, the forecasts reported by the ACARR model are not objective.

This assumption was tested by the Granger causality test on the upward range and the downward range. The Granger causality test is performed based on the following linear equation:

here p is the order of the lag. The null hypothesis of the test is that does not Granger cause , it corresponds to . If the null hypothesis was rejected, it can be concluded that Granger causes .

* 1. **Feedback ACARR model (FACARR)**

In the previous paragraph we have got that there is significant Granger causality between the upward range and the downward range, especially from the downward range to the upward range. To include this cross-interdependence the following ACARR model was proposed:

In contrast to the ACARR model, this one allows interdependence between the upward range and the downward range. Hence, this model is more flexible for capturing the asymmetry in the price range.

This new model is called the feedback ACARR model (FACARR). In this model the upward range and downward range are used as both the explanatory variables and the explained variables.

* 1. **What if the FCARR assumptions are removed?**

We return to the ACARR model that we try to improve.

* 1. **What data is needed to be collected to test the FCARR model?**

To test the model we need to collect stock data from <https://finance.yahoo.com/>

In our opinion it is very applicable to use the model on real life data for everyday forecast of the volatility of the stock prices.

1. **Empirical studies**
   1. **Granger causality testing results**

Firstly, the data of the upward range and the downward range was filtered with the ACARR (1, 1) model with exponential distribution. The filtered data can reduce or eliminate the clustering effect in the upward range and the downward range. Eventually, the results show that the null hypothesis that there is no Granger causality between the upward range and the downward range is rejected.

The results show significant cross-interdependence between the upward range and the downward range. Therefore, the implied assumption in the ACARR model was contradicted. The lag length selection for the Granger causality test is chosen using the Bayesian information criterion.

* 1. **In-sample and out-of-sample forecasts**

From the results in 3.1 we get that the implied assumption in the ACARR model is invalid. Nevertheless, we still do not know if the cross-interdependence between the upward range and the downward range is important for predicting financial volatility. We will compare the forecasting power of the ACARR model and the FACARR model, which takes into consideration the cross-interdependence, to look if a new FACARR model has better forecasting power.

1. **Conclusion**
   1. **Conclusions of the authors**

The authors of the paper concluded that:

* The results of empirical studies, which was performed on a variety of stock indices, show that the forecasting power of the ACARR model can be significantly improved if the cross-interdependence is included.
* FCARR model performs better than the ACARR model in forecasting volatility.
  1. **How according to the authors the model can be improved?**

The relative performance of the FACARR model to the GARCH like models in volatility forecasting also needs more investigations to understand how useful the model is and what limitation of the model is.

In the paper only the empirical evidence was presented and it showed that there is significant cross-interdependence between the upward range and the downward range. The economic reasons behind this interdependence are largely unexplored and remain unknown.

* 1. **Our opinion regarding improving the FACARR model**

We suggest to improve the model by applying “Ridge Regression” to estimate the model parameters instead of a “linear regression” the authors used, we suppose that using a different type of regression can improve the parameters estimation precision and improve the forecasting power of the model.

We tested our assumption. The empirical studies show that if we use “Ridge Regression” the root mean squared error (RMSE) three times less than RMSE of the “linear regression” in out-of-sample forecast.

* 1. **What is the main contribution of the article?**

The main contribution of this article is by significantly improving the already existing ACARR model and confirming the assumptions regarding the cross-interdependence between the up/down prices of the assets.